The Multi-State Constraint Kalman Filter

- A Filtering Approach for Visual-Inertial Navigation -

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Simultaneous Localization and Mapping (SLAM)

What SLAM answers…

Where am I?

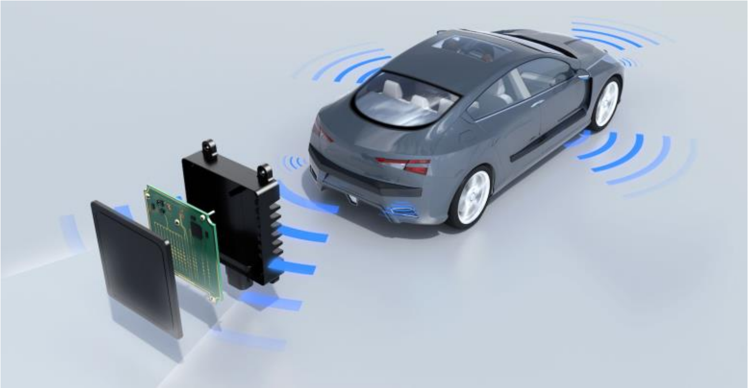
What does the world look like?

What SLAM enables…

Where should I go?

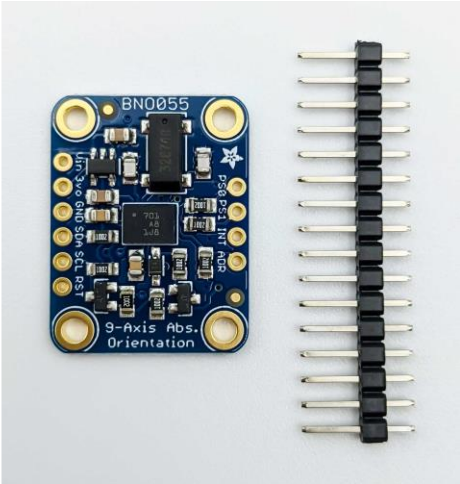
Sensors allow robots to localize and perceive the environment.

LiDAR



Radar

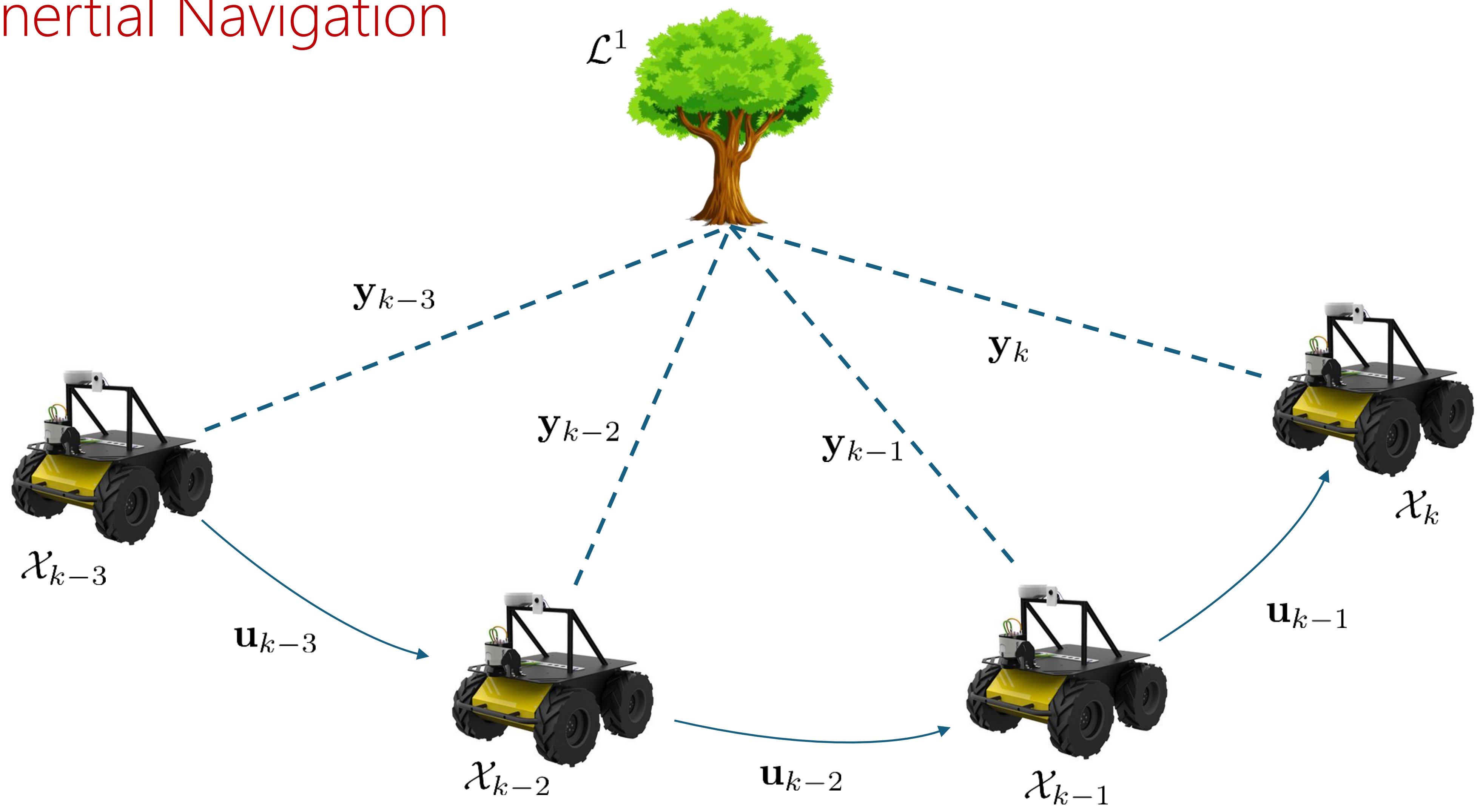
Visual-Inertial



Inertial measurement unit (IMU)

Cameras

Visual-Inertial Navigation



Goal: Infer the robot state $\mathcal{X}_{k-3:k}$ and landmark position $\mathcal{L}^1$ given:

- inertial measurements $\mathbf{u}_{k-3:k-1}$,
- visual feature measurements $\mathbf{y}_{k-3:k}$.

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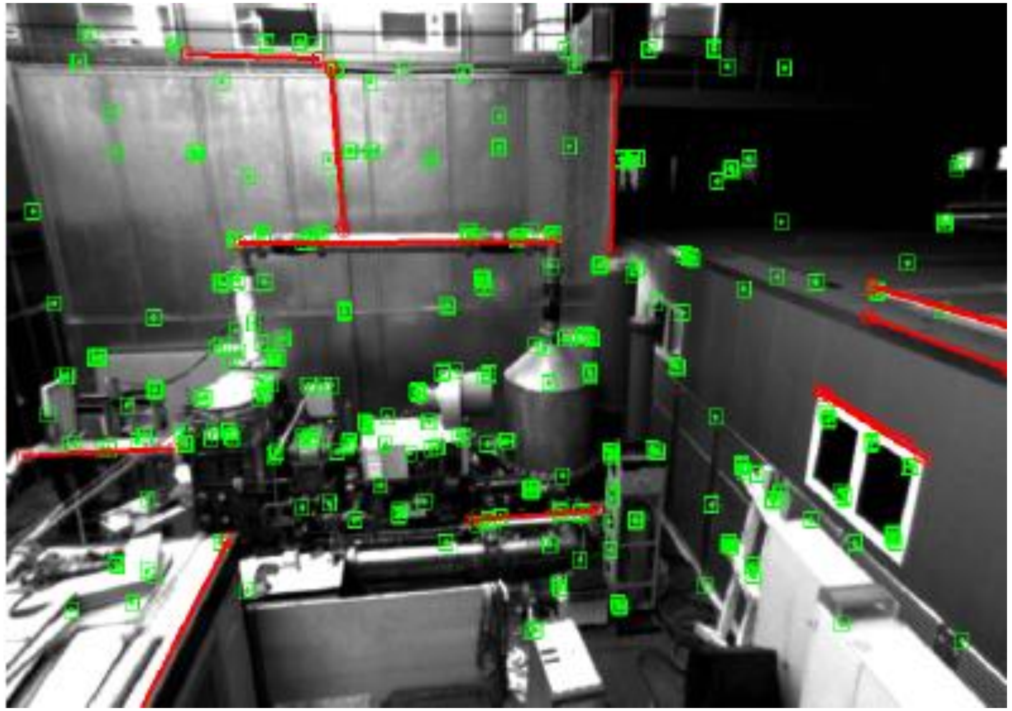
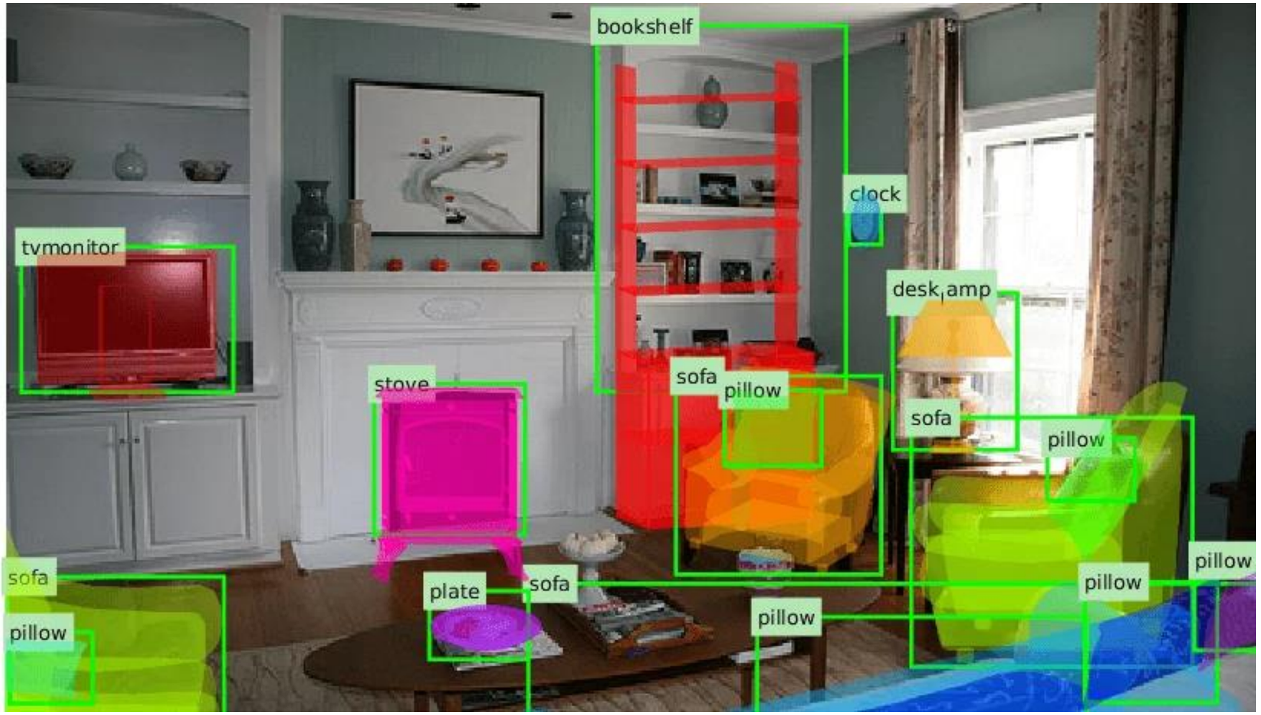
What do we mean by features?

Features: distinctive visual information in the environment.

- Points, lines, planes, objects…



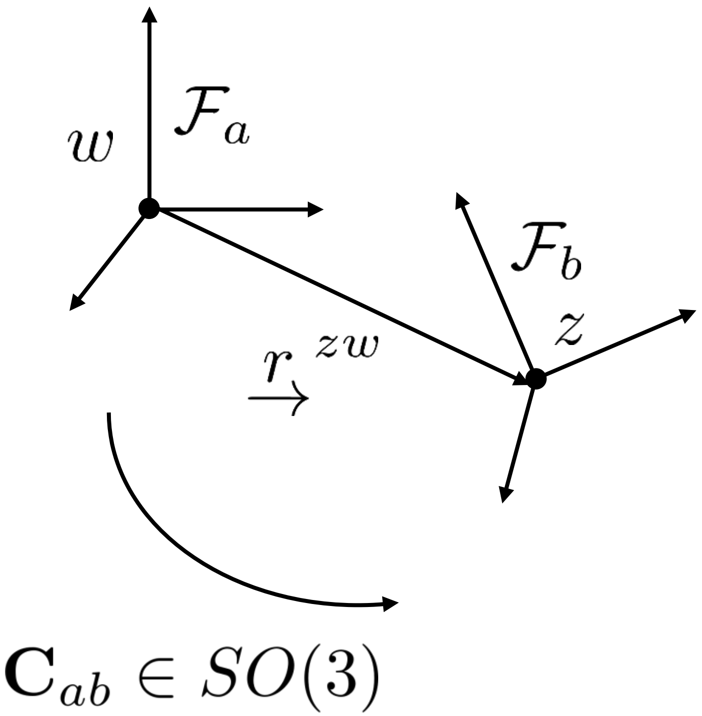
Points and lines extracted from image



Objects extracted from image

IMU Measurements

- IMUs consist of a **rate gyro** and an **accelerometer** and measure the acceleration and angular velocity of a vehicle, with additive bias and noise.



True angular velocity

Gyro: $\mathbf{u}^g_{b_k} = \boldsymbol{\omega}^{b_k a}_{b_k} + \mathbf{b}^g_{b_k} + \mathbf{w}^g_{b_k}$

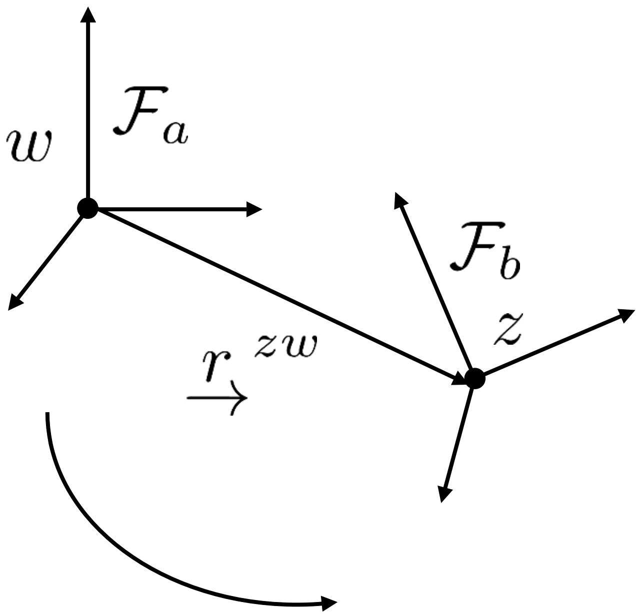
Accelerometer: $\mathbf{u}^a_{b_k} = \mathbf{C}^{\mathsf{T}}_{ab_k}\left(\mathbf{a}^{z_k w/a/a}_a - \mathbf{g}_a\right) + \mathbf{b}^a_{b_k} + \mathbf{w}^a_{b_k}$

True acceleration

The IMU Kinematics

- The IMU state is the **orientation**, **velocity** and **position** of the IMU, and the IMU **biases**.

$$\mathcal{X}^{\mathrm{IMU}} = \left(\mathbf{C}_{ab}, \mathbf{v}_a^{zw/a}, \mathbf{r}_a^{zw}, \mathbf{b}_b^g, \mathbf{b}_b^a \right).$$



$$\mathbf{C}_{ab} \in SO(3)$$

Continuous-Time Process Model

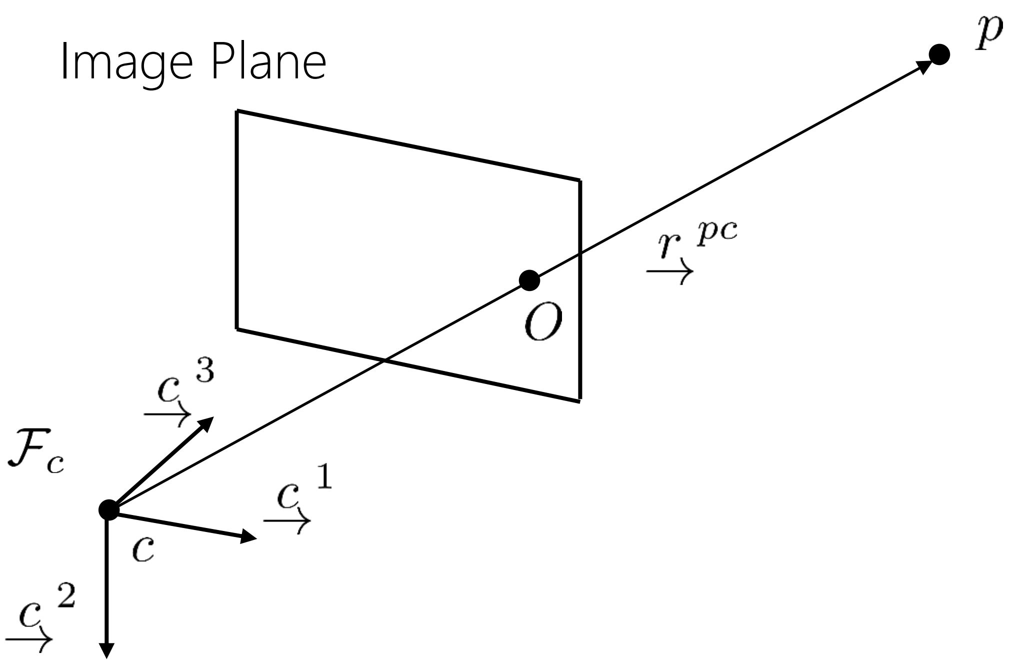
$$\begin{aligned}
\dot{\mathbf{C}}_{ab} &= \mathbf{C}_{ab} \left(\mathbf{u}_b^g - \mathbf{b}_b^g - \mathbf{w}_b^g \right)^{\times}, \\
\dot{\mathbf{v}}_a^{zw/a} &= \mathbf{C}_{ab} \left(\mathbf{u}_b^a - \mathbf{b}_b^a - \mathbf{w}_b^a \right) + \mathbf{g}_a, \\
\dot{\mathbf{r}}_a^{zw} &= \mathbf{v}_a^{zw/a}, \\
\dot{\mathbf{b}}_b^g &= \mathbf{w}_b^{b_g}, \\
\dot{\mathbf{b}}_b^a &= \mathbf{w}_b^{b_a}.
\end{aligned}$$

Discrete-Time Process Model

$$\begin{aligned}
&\mathcal{X}_k^{\mathrm{IMU}} = f_{k-1}\left(\mathcal{X}_{k-1}^{\mathrm{IMU}}, \mathbf{u}_{k-1} \right) \oplus \mathbf{w}_{k-1}, \\
&\mathbf{w}_{k-1} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{Q}_{k-1} \right)
\end{aligned}$$

Visual Feature Measurements

- Cameras model a mapping between the 3D world and a 2D image.
- Commonly modelled as **central projection** – project points in 3D space onto a plane called the image plane.



$$\mathbf{r}_c^{pc} = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix}^\mathsf{T} \in \mathbb{R}^3$$

Pinhole Camera Model

$$\begin{bmatrix} x_n \\ y_n \end{bmatrix} = \mathbf{g}\left(\mathbf{r}_c^{pc}\right) = \begin{bmatrix} r_1/r_3 & r_2/r_3 \end{bmatrix}.$$

Normalized image coordinates of observation

Visual Feature Measurements

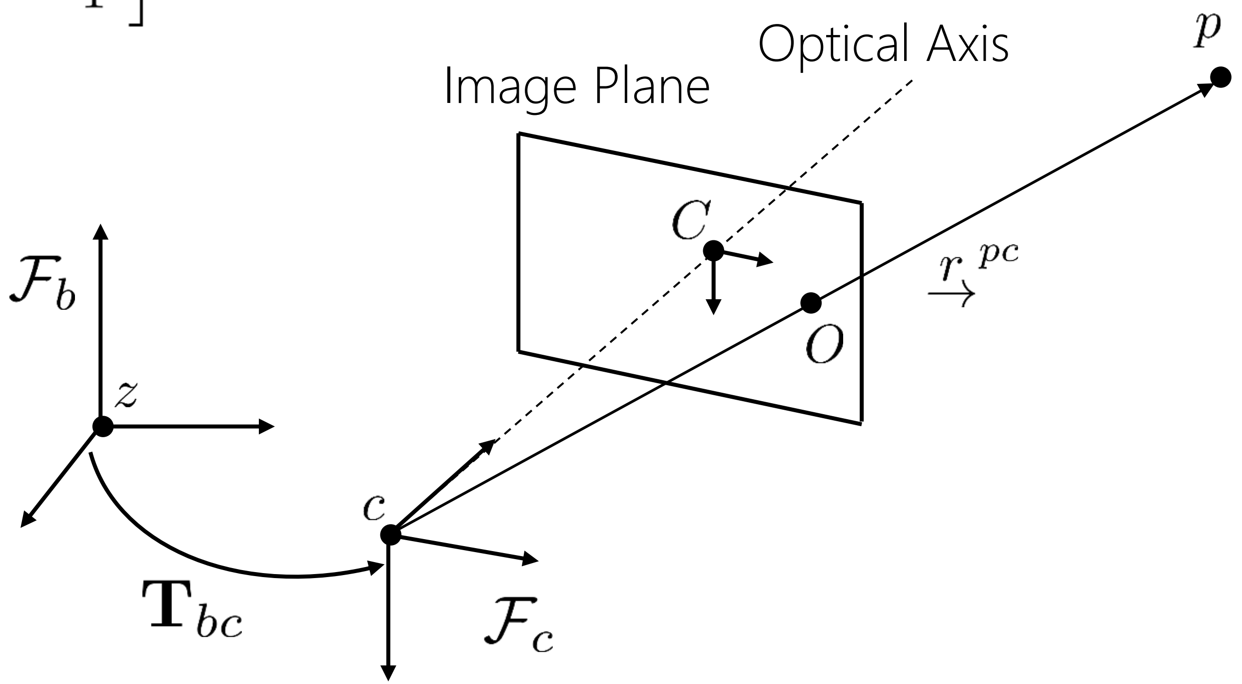
- We need to write our measurement model in terms of the states of interest.
- This can be done using the camera/IMU **extrinsic parameters.**

$$\mathbf{T}_{bc} = \begin{bmatrix} \mathbf{C}_{bc} & \mathbf{r}_b^{cz} \\ \mathbf{0} & 1 \end{bmatrix} \in SE(3)$$

Full measurement model

$$\mathbf{r}_c^{pc} = \mathbf{C}_{bc}^{\mathsf{T}} \left(\mathbf{C}_{ab}^{\mathsf{T}} \left(\mathbf{r}_a^{pw} - \mathbf{r}_a^{zw} \right) - \mathbf{r}_b^{cz} \right)$$

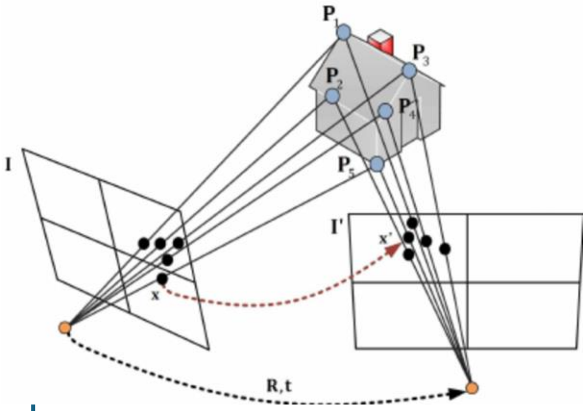
$$\begin{bmatrix} x_n \\ y_n \end{bmatrix} = \mathbf{g}\left(\mathbf{r}_c^{pc}\right) = \begin{bmatrix} r_1/r_3 & r_2/r_3 \end{bmatrix}$$



Visual-Inertial Navigation Approaches

Loosely-Coupled Fusion

Step 1: Determine relative poses between images using two-view geometry.



Step 2: Fuse relative pose information with IMU (filters, sliding window filters, etc.)

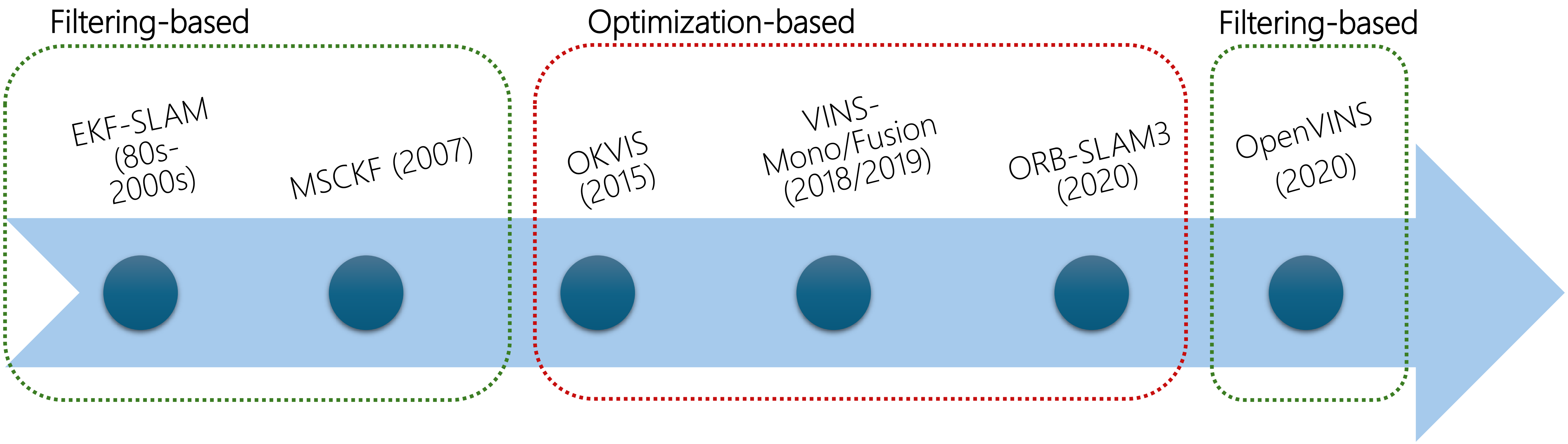
Computationally efficient!

Tightly-Coupled Fusion

Directly fuse camera and IMU data within a single process.

Higher accuracy, higher computational cost.

Tightly-Coupled Visual-Inertial Algorithms



- **Filtering-based** algorithms use measurements a single time to estimate the state.
- **Optimization-based** algorithms perform iterative minimization over a window of states.

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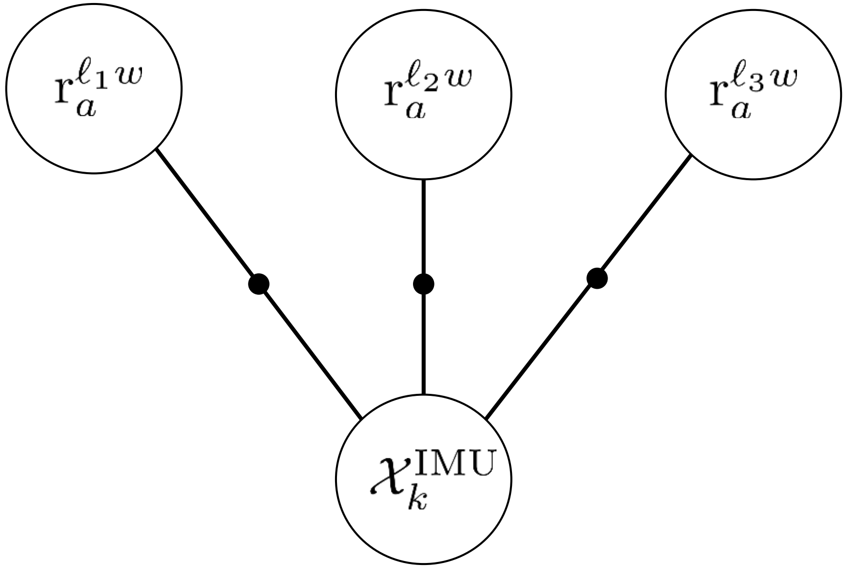
Classic EKF-SLAM

- EKF state includes current IMU state and landmark positions.

$$\mathcal{X}_k = \left(\mathcal{X}_k^{\mathrm{IMU}}, \mathbf{r}_a^{\ell_1 w}, \ldots, \mathbf{r}_a^{\ell_n w}\right)$$

IMU state Landmark position



- State covariance can then be partitioned as

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}^{\mathcal{X}\mathcal{X}} & \mathbf{P}^{\mathcal{X}\mathcal{L}} \\ \mathbf{P}^{\mathcal{L}\mathcal{X}} & \mathbf{P}^{\mathcal{L}\mathcal{L}} \end{bmatrix}$$

- The IMU state is the orientation, velocity and position of the IMU, and the IMU biases.

$$\mathcal{X}^{\mathrm{IMU}} = \left(\mathbf{C}_{ab}, \mathbf{v}_a^{zw/a}, \mathbf{r}_a^{zw}, \mathbf{b}_b^{g}, \mathbf{b}_b^{a}\right).$$

Classic EKF-SLAM – Prediction Step

- Reduced form of the typical EKF prediction step due to the structure of the problem.

- IMU process model and process model Jacobian are given by

$$\mathcal{X}_k^{\mathrm{IMU}} = f_{k-1}\left(\mathcal{X}_{k-1}, \mathbf{u}_{k-1}\right) \oplus \mathbf{w}_{k-1}, \qquad \mathbf{w}_{k-1} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{Q}_{k-1}\right)$$

$$\mathbf{F}_{k-1}^{\mathrm{IMU}} = \left.\frac{Df\left(\mathcal{X}^{\mathrm{IMU}}, \mathbf{u}_{k-1}\right)}{D\mathcal{X}^{\mathrm{IMU}}}\right|_{\hat{\mathcal{X}}_{k-1}}$$

- Landmarks are assumed constant!

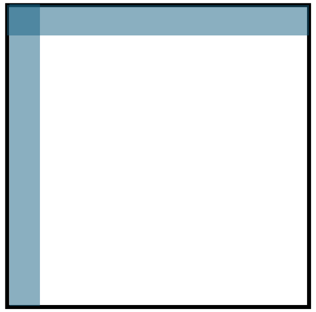
$$\dot{\mathbf{r}}_a^{\ell_i w} = \mathbf{0}, \quad i = 1, \ldots, m$$

- The EKF covariance propagation has the form

Updated parts of the covariance

$$\check{\mathbf{P}}_k = \mathbf{F}_{k-1}^{\mathrm{IMU}} \hat{\mathbf{P}}_{k-1} \mathbf{F}_{k-1}^{\mathrm{IMU}^{\mathsf{T}}} + \mathbf{Q}_{k-1}$$

$$\check{\mathbf{P}}_k = \begin{bmatrix} \check{\mathbf{P}}_k^{\mathcal{X}\mathcal{X}} & \mathbf{F}_{k-1}^{\mathrm{IMU}} \hat{\mathbf{P}}_{k-1}^{\mathcal{X}\mathcal{L}} \\ \hat{\mathbf{P}}_{k-1}^{\mathcal{L}\mathcal{X}} \mathbf{F}_{k-1}^{\mathrm{IMU}^{\mathsf{T}}} & \hat{\mathbf{P}}_{k-1}^{\mathcal{L}\mathcal{L}} \end{bmatrix}$$



Classic EKF-SLAM – Correction Step

- The EKF correction step can also be modified to exploit sparsity.
- Each measurement links **one** IMU state to **one** landmark state.

Measurement Model: $\mathbf{y}_{jk} = \mathbf{g}\left(\mathcal{X}_k^{\text{IMU}}, \mathbf{r}_a^{\ell_j w}\right) + \mathbf{v}_{jk}$

Measurement Model Jacobian: $\mathbf{H}_{jk} = \begin{bmatrix} \mathbf{H}_k & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H}_j & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}$

$$\mathbf{H}_k = \left.\frac{D\mathbf{g}(\mathcal{X}, \mathbf{r}_a^{\ell w})}{D\mathcal{X}}\right|_{\hat{\mathcal{X}}_k^{\text{IMU}}}, \qquad \mathbf{H}_j = \left.\frac{D\mathbf{g}(\mathcal{X}, \mathbf{r}_a^{\ell w})}{D\mathbf{r}_a^{\ell w}}\right|_{\hat{\mathbf{r}}_a^{\ell w}}$$

- The computation of the innovation is sparse, but the Kalman gain is **dense**.

$$\hat{\mathcal{X}} = \check{\mathcal{X}} \oplus \mathbf{K}\mathbf{z}$$
$$\mathbf{P}_k = \left(\mathbf{1} - \mathbf{K}\mathbf{H}_{jk}\right)\check{\mathbf{P}}_k$$

$\longrightarrow$ Entire state and covariance must be updated for each landmark observation!

Complexity $\mathcal{O}\left(n^2\right)$ per measurement.

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The Problem with EKF-SLAM

- The computational complexity of the EKF scales worst-case **cubically** with the number of landmarks.
- The EKF quickly becomes computationally intractable for large maps.
- Possible remedies:
  - decouple the estimation problem into a series of smaller submaps [1],
  - utilize the Extended Information Filter [2].

- **Is there a way to remove the landmarks from the state vector?**

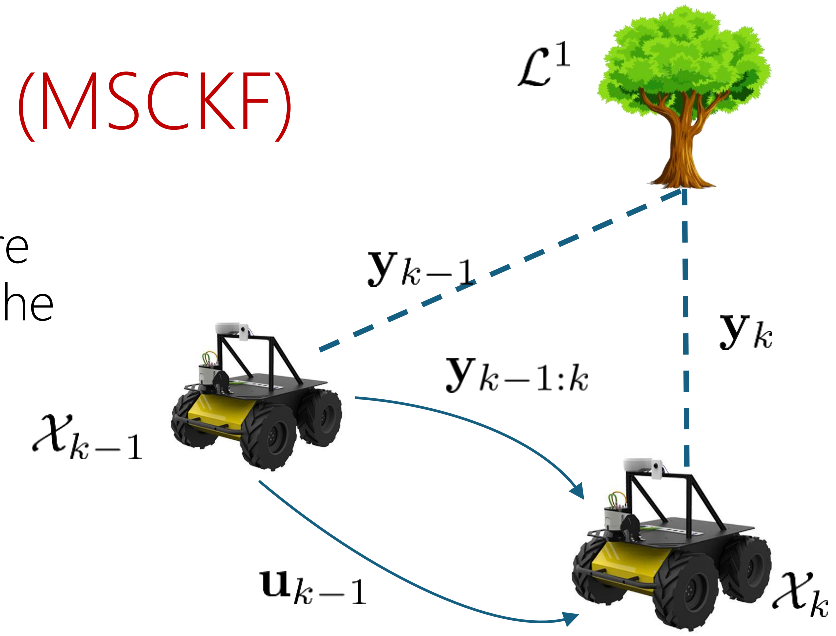
[1] J. Leonard and H. Feder, "Decoupled Stochastic Mapping," IEEE Journal of Oceanic Engineering, vol. 26, no. 4, pp. 561–571, 2001

[2] S. Thrun, Y. Liu, D. Koller, A. Ng, Z. Ghahramani, and H. Durrant-Whyte, "Simultaneous Localization and Mapping with Sparse Extended Information Filters," International Journal of Robotics Research, 2004

The Multi-State Constraint Kalman Filter (MSCKF)

- Key idea: Express the constraint imposed by a static feature observed from multiple camera poses, **without** including the feature position in the state vector.

- The MSCKF state includes m IMU poses.



MSCKF State

$$\mathcal{X}_k = (\mathcal{X}_k^{\mathrm{IMU}}, \mathcal{X}_{k-1}^p, \mathcal{X}_{k-2}^p, \cdots, \mathcal{X}_{k-m}^p)$$

$$\mathcal{X}_k^p = (\mathbf{C}_{ab_k}, \mathbf{r}_a^{z_k w})$$

EKF State

$$\mathcal{X}_k = (\mathcal{X}_k^{\mathrm{IMU}}, \mathbf{r}_a^{\ell_1 w}, \cdots, \mathbf{r}_a^{\ell_m w})$$

- The MSCKF is still an EKF-based estimator.
  - Follows a predict-correct structure!

MSCKF Overview

- **Step 1 – Prediction:** Propagate the state forward using the IMU measurements.
- **Step 2 – State Augmentation:** At each new image, augment the state and covariance with a copy of the current IMU pose.
- **Step 3 – Image Processing:** Extract and match features on the image.
- **Step 4 – Correction:** when an update step is triggered, utilize *all* measurements of a given landmark to perform the EKF correction step and update the state.

MSCKF Overview – Prediction Step

- Similar propagation step to EKF-SLAM due to covariance partitioning!

$$\mathcal{X}_k = \left(\mathcal{X}_k^{\text{IMU}}, \mathcal{X}_{k-1}^p, \mathcal{X}_{k-2}^p, \cdots, \mathcal{X}_{k-m}^p\right)$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}^{\mathcal{X}\mathcal{X}} & \mathbf{P}^{\mathcal{X}\mathcal{P}} \\ \mathbf{P}^{\mathcal{P}\mathcal{X}} & \mathbf{P}^{\mathcal{P}\mathcal{P}} \end{bmatrix}$$

IMU State Covariance: $\mathbf{P}^{\mathcal{X}\mathcal{X}} \in \mathbb{R}^{15\times 15}$

IMU Clone Covariances: $\mathbf{P}^{\mathcal{P}\mathcal{P}} \in \mathbb{R}^{6m\times 6m}$

- Current IMU state is propagated forward using process model,

$$\mathcal{X}_k^{\text{IMU}} = f_{k-1}\left(\mathcal{X}_{k-1}, \mathbf{u}_{k-1}\right).$$

- Covariance is propagated forwards as

$$\check{\mathbf{P}}_k = \mathbf{F}_{k-1}^{\text{IMU}} \hat{\mathbf{P}}_{k-1} \mathbf{F}_{k-1}^{\text{IMU}^\mathsf{T}} + \mathbf{Q}_{k-1}$$

$$\check{\mathbf{P}}_k = \begin{bmatrix} \check{\mathbf{P}}_k^{\mathcal{X}\mathcal{X}} & \mathbf{F}_{k-1}^{\text{IMU}} \hat{\mathbf{P}}_{k-1}^{\mathcal{X}\mathcal{L}} \\ \hat{\mathbf{P}}_{k-1}^{\mathcal{L}\mathcal{X}} \mathbf{F}_{k-1}^{\text{IMU}^\mathsf{T}} & \hat{\mathbf{P}}_{k-1}^{\mathcal{L}\mathcal{L}} \end{bmatrix}$$

MSCKF Overview – State Augmentation

- When an image is received, augment the state and covariance with a copy of the current IMU pose.

$$\hat{\mathcal{X}} = \left(\hat{\mathcal{X}}_k^{\mathrm{IMU}}\right) \longrightarrow \hat{\mathcal{X}}_k^{+} = \left(\hat{\mathcal{X}}_k^{\mathrm{IMU}}, \hat{\mathcal{X}}_k^{p}\right)$$

- This operation is called **stochastic cloning.**

Current state

$$\mathcal{V} = c\left(\mathcal{X}\right)$$

New variable

Cloning function

- Augment state and covariance as

$$\hat{\mathcal{X}}^{+} = \left(\hat{\mathcal{X}}, \quad c\left(\hat{\mathcal{X}}\right)\right),$$

$$\hat{\mathbf{P}}^{+} = \begin{bmatrix} \hat{\mathbf{P}} & \hat{\mathbf{P}}\mathbf{C}^{\mathsf{T}} \\ \mathbf{C}\hat{\mathbf{P}} & \mathbf{C}\hat{\mathbf{P}}\mathbf{C}^{\mathsf{T}} \end{bmatrix},$$

$$\mathbf{C} = \left.\frac{Dc\left(\mathcal{X}\right)}{D\mathcal{X}}\right|_{\hat{\mathcal{X}}}.$$

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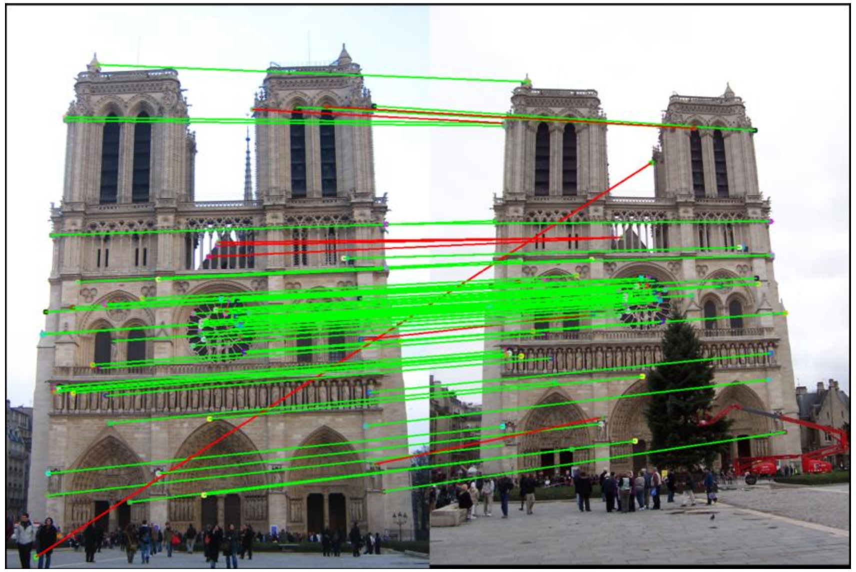
MSCKF Overview – Image Processing

- Extract and match features on the image.
- Common techniques include utilizing **optical flow**, or **descriptor matching**.

Optical flow finds apparent motion between images.



Descriptor matching tries to find similar features in "descriptor" space.

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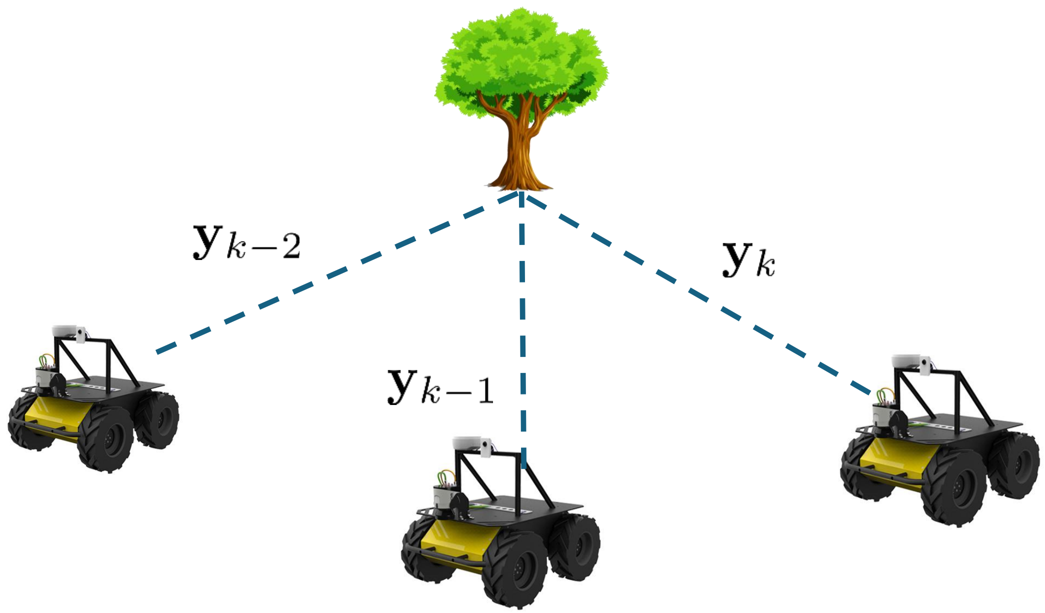
[Images taken from](http://jevois.org/tutorials/ProgrammerInvFlowLK.html)
http://jevois.org/tutorials/ProgrammerInvFlowLK.html
https://faculty.cc.gatech.edu/~hays/compvision2018/proj2/

MSCKF Overview – Correction Step

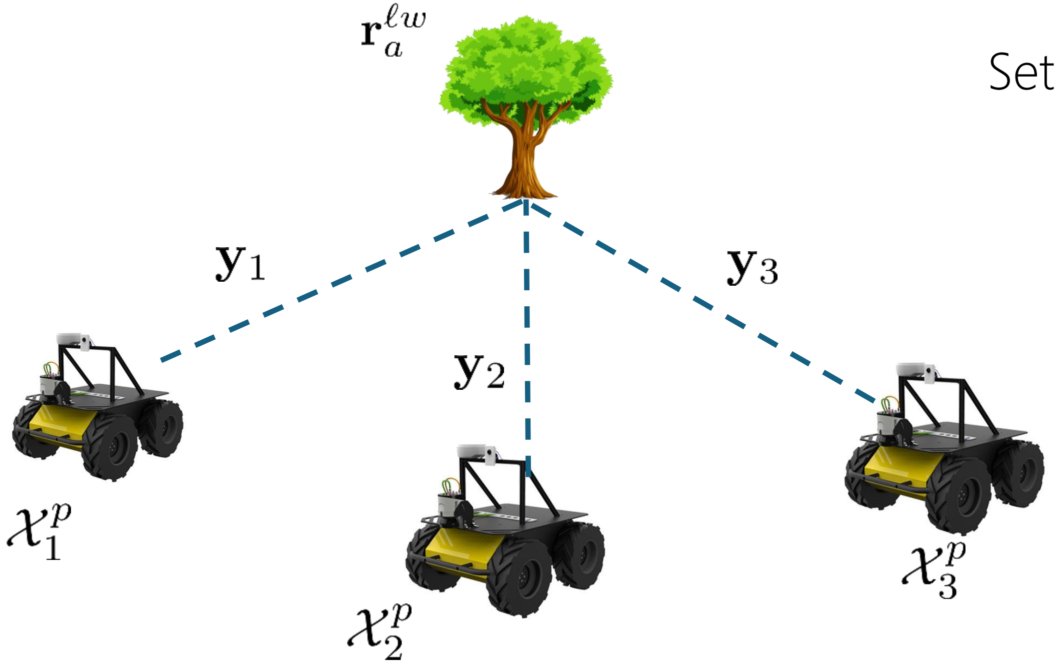
- **Step 4 – Correction:** when an update step is triggered, utilize *all* measurements of a given feature to perform the EKF correction step and update the state.



- **First question:** How do we utilize all measurements of a given feature to correct the MSCKF state?
- **Second question:** When do we trigger the correction step?

MSCKF Overview – The Measurement Model

- Consider the measurements of a single feature, $\mathbf{r}_a^{\ell w}$, observed from a set of M_j robot poses.



Set of robot poses: $\{\mathcal{X}_i^p\}, i \in \mathcal{S}_j$

Observations: $\mathbf{y}_i = \mathbf{g}\left(\mathbf{r}_{c_i}^{\ell c_i}\right) + \mathbf{v}_k, \quad \mathbf{v}_k \sim \mathcal{N}\left(\mathbf{0}, \mathbf{R}_k\right).$

Feature resolved in camera frame:

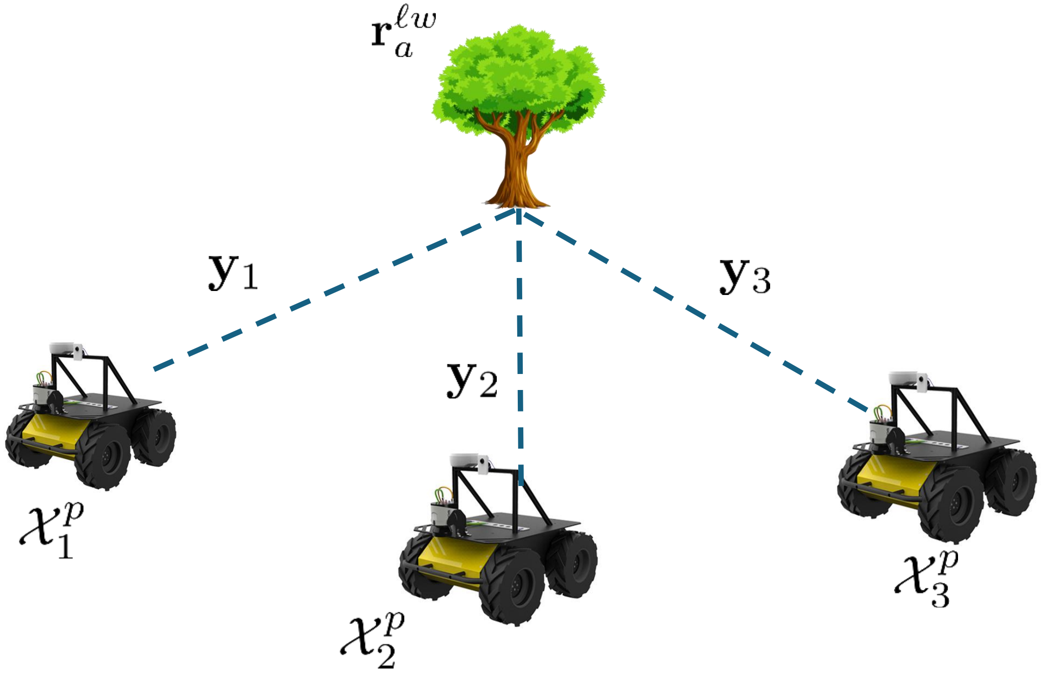
$$\mathbf{r}_{c_i}^{\ell c_i} = \mathbf{C}_{ac_i}^{\mathsf{T}}\left(\boxed{\mathbf{r}_a^{\ell w}} - \mathbf{r}_a^{c_i w}\right)$$

Inertial landmark position

- We want to be able to predict the measurements from the state estimates.

MSCKF Overview – Solving for the Landmark Position

- We need an we need an estimate of $\mathbf{r}_a^{\ell w}$!
- Conduct batch estimation to solve for an estimate of the landmark position as



$$\hat{\mathbf{r}}_a^{\ell w} = \arg\max_{\mathbf{r}_a^{\ell w}} p\left(\mathbf{r}_a^{\ell w} | \mathbf{y}_i, \hat{\mathcal{X}}_i^p\right), \quad i \in \mathcal{S}_j$$

Predict measurement

$$\hat{\mathbf{r}}_{c_i}^{\ell c_i} = \hat{\mathbf{C}}_{ac_i}^{\mathsf{T}} \left(\hat{\mathbf{r}}_a^{\ell w} - \hat{\mathbf{r}}_a^{c_i w}\right)$$

$$\hat{\mathbf{y}}_i = \mathbf{g}\left(\hat{\mathbf{r}}_{c_i}^{\ell c_i}\right)$$

- IMU poses are treated as known constants and are held **fixed** in the batch problem.

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MSCKF Overview – The Measurement Model

- Measurement residual can now be computed using the estimated feature position as

$$\mathbf{z}_i = \mathbf{y}_i - \mathbf{g}\left(\hat{\mathbf{r}}_{c_i}^{\ell c_i}\right)$$

- Linearize residual:

$$\delta \mathbf{r}_a^{\ell w} = \mathbf{r}_a^{\ell w} - \hat{\mathbf{r}}_a^{\ell w}$$

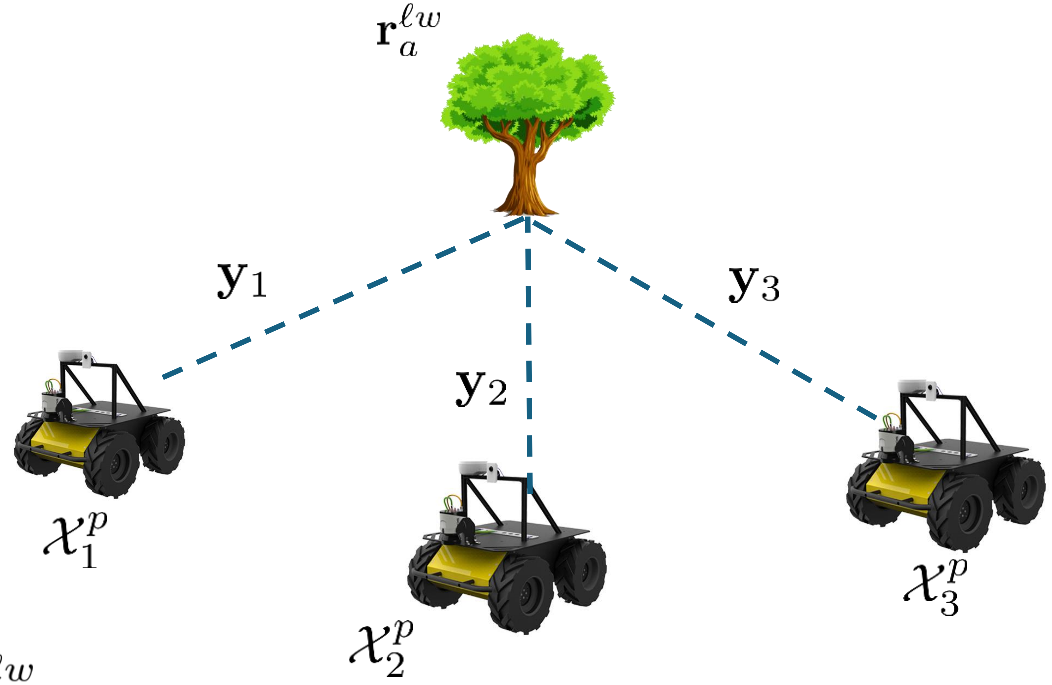
$$\delta \mathbf{z}_i = \mathbf{H}^{\mathcal{X}_i^p} \delta \boldsymbol{\xi}_i^p + \mathbf{H}^{\ell} \delta \mathbf{r}_a^{\ell w}$$

Pose Jacobian

Landmark Jacobian

- Stack all residuals of a single feature:

$$\mathbf{z} = \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \vdots \\ \mathbf{z}_{M_j} \end{bmatrix} \xrightarrow{\text{Stack linearizations}} \delta \mathbf{z} = \mathbf{H}^{\mathcal{X}^p} \delta \boldsymbol{\xi}^p + \mathbf{H}^{\ell} \delta \mathbf{r}_a^{\ell w}$$



- Cannot directly use this in the EKF correction step…

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MSCKF Overview – MSCKF Nullspace Projection

- The MSCKF "secret sauce": project original residual onto the left **nullspace** of the feature Jacobian.

Original linearized residual: $\delta \mathbf{z} = \mathbf{H}^{\mathcal{X}^p} \delta \boldsymbol{\xi}^p + \mathbf{H}^{\ell} \delta \mathbf{r}_a^{\ell w}, \quad \mathbf{H}^{\ell} \in \mathbb{R}^{2M_j \times 3}$

- Nullspace of the feature Jacobian is spanned by the columns of $\mathbf{N}$,

$$\mathbf{N} = \begin{bmatrix} \mathbf{n}_1 & \cdots & \mathbf{n}_{2M_j-3} \end{bmatrix} \in \mathbb{R}^{2M_j \times (2M_j-3)}, \qquad \mathbf{H}^{\ell^{\mathsf{T}}} \mathbf{n}_i = \mathbf{0}.$$

$$\mathbf{N}^{\mathsf{T}} \mathbf{H}^{\ell} = \mathbf{0}$$

$$\downarrow$$

$$\mathbf{N}^{\mathsf{T}} \delta \mathbf{z} = \mathbf{N}^{\mathsf{T}} \mathbf{H}^{\mathcal{X}^p} \delta \boldsymbol{\xi}^p + \cancelto{0}{\mathbf{N}^{\mathsf{T}} \mathbf{H}^{\ell} \delta \mathbf{r}_a^{\ell w}}$$

New residual definition: $\mathbf{z}' = \mathbf{N}^{\mathsf{T}} \mathbf{z} \in \mathbb{R}^{2M_j-3}$

- SVD can be used to compute the left nullspace.

$$\mathbf{H}^{f} = \mathbf{U} \boldsymbol{\Sigma} \mathbf{V}^{\mathsf{T}}$$

Take columns corresponding to 0 singular values

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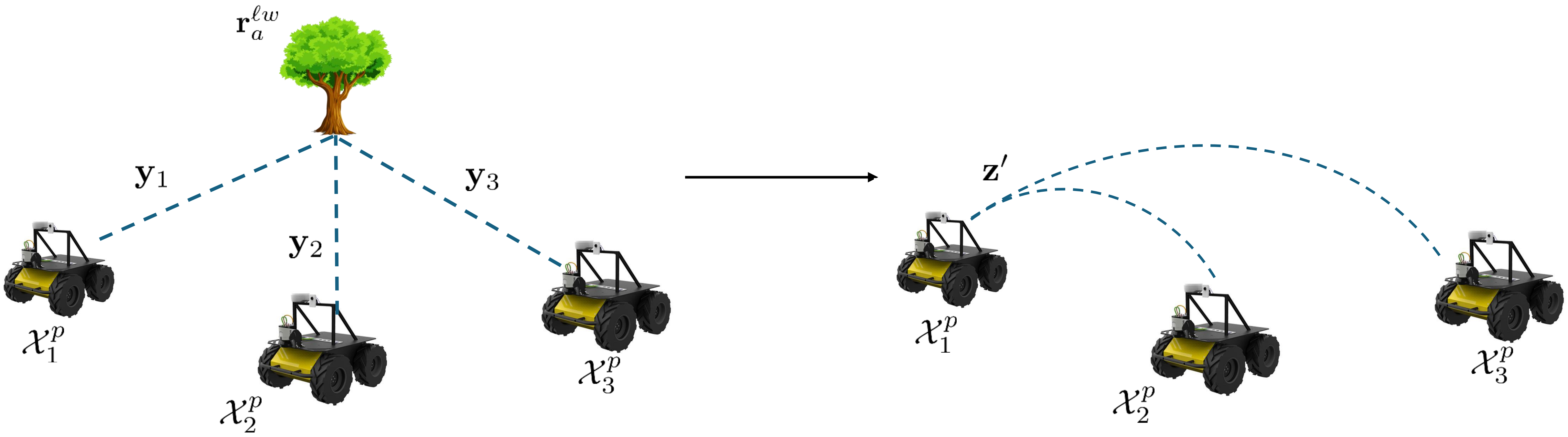
MSCKF Overview – MSCKF Nullspace Projection

- New linearized residual is **independent** of the errors in the feature positions.
  - We can now use this residual and linearization in an EKF update step!

- A constraint is defined between all the camera poses from which a given feature was observed.



MSCKF Overview – EKF Update Trigger

- Now we've showed how to express the constraint imposed by observing a landmark from multiple camera poses.

- An EKF updated using this constraint is triggered by one of two conditions:
    1. Once a feature has lost tracking, all measurements of that feature are used in an EKF update.

    2. Once a maximum allowable number of IMU poses ($N_{\max}$) in the state vector has been reached, remove a number of these camera poses and utilize all measurements from these selected poses.

Example: if $N_{\max} = 5$

$$\mathcal{X}_k = (\mathcal{X}_k^{\mathrm{IMU}}, \mathcal{X}_{k-1}^p, \mathcal{X}_{k-2}^p, \mathcal{X}_{k-3}^p, \mathcal{X}_{k-4}^p, \mathcal{X}_{k-5}^p)$$

Remove from state vector and utilize all measurements.

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MSCKF Overview – EKF Update Equations

- Consider a situation in which the constraints from L features, selected from the two previous criteria, must be processed.

- Compute the (projected) residual and linearization for each feature

$$\mathbf{z}'_j = \mathbf{N}_j^{\mathsf{T}} \mathbf{z}_j \in \mathbb{R}^{2M_j - 3}, \quad j = 1, \dots, L.$$

$$\delta \mathbf{z}'_j = \mathbf{N}_j^{\mathsf{T}} \delta \mathbf{z}_j = \mathbf{N}_j^{\mathsf{T}} \mathbf{H}^{\mathcal{X}^p} \delta \boldsymbol{\xi}$$

- Stack all residuals and linearizations as

$$\begin{aligned} \mathbf{z}' &= \begin{bmatrix} \mathbf{z}_1^{\mathsf{T}} & \cdots & \mathbf{z}_L^{\mathsf{T}} \end{bmatrix}^{\mathsf{T}} \\ \delta \mathbf{z}' &= \mathbf{H} \delta \boldsymbol{\xi} \end{aligned}$$

- Residual dimension can now be quite large – size of residual is given by

$$d = \sum_{j=1}^{L} (2M_j - 3)$$

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MSCKF Overview – EKF Update Equations

- QR decomposition saves the day (as usual).
- If the dimension of the residual is larger than the state dimension, decompose the full Jacobian as

$$\begin{aligned}\mathbf{H} &= \begin{bmatrix}\mathbf{Q}_1 & \mathbf{Q}_2\end{bmatrix}\begin{bmatrix}\mathbf{H}^r \\ \mathbf{0}\end{bmatrix} \\ &= \mathbf{Q}_1\mathbf{H}^r\end{aligned}$$

- Project the residual onto this basis vectors of the range of $\mathbf{H}$ as

$$\mathbf{z}^r = \mathbf{Q}_1^\mathsf{T}\mathbf{z}$$

$$\downarrow$$

$$\begin{bmatrix}\mathbf{Q}_1^\mathsf{T}\delta\mathbf{z} \\ \mathbf{Q}_2^\mathsf{T}\delta\mathbf{z}\end{bmatrix} = \begin{bmatrix}\mathbf{H}^r \\ \mathbf{0}\end{bmatrix}\delta\boldsymbol{\xi}$$

$$\delta\mathbf{z}^r = \mathbf{H}^r\delta\boldsymbol{\xi}$$

- This reduced residual and Jacobian can now be used in the EKF update!

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MSCKF Summary

- The computational complexity of the MSCKF is **linear** in the number of features, and at worst **cubic** in the number of poses that are included in the state vector.

- The MSCKF can be thought of as a hybrid between a sliding window filter and the standard EKF.
  - Both maintain historical poses in the state vector.

$$\mathcal{X}_k = \left(\mathcal{X}_k^{\mathrm{IMU}}, \mathcal{X}_{k-1}^{p}, \mathcal{X}_{k-2}^{p}, \cdots, \mathcal{X}_{k-m}^{p}\right)$$

  - The MSCKF still only linearizes the measurement model a single time!

MSCKF Nullspace Projection Dimentions

- Matrix dimensions of involved quantities in nullspace projection:

$$\mathbf{H}^{\ell} \in \mathbb{R}^{2n \times 3}$$

$$\operatorname{rank}(\mathbf{H}^{\ell}) \leq \min(2n, 3) = 3$$

$$\operatorname{nullity}\left(\mathbf{H}^{\ell^{\mathsf{T}}}\right) = 2n - 3$$

$$\mathbf{N} \in \mathbb{R}^{2n \times (2n-3)}$$

$$\mathbf{z}' = \mathbf{N}^{\mathsf{T}} \mathbf{H}^{\ell} \in \mathbb{R}^{2n \times 3}$$